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The Demographic Effects of Increasing Anthropogenic Noise on Mysticeti Whales

Iván E. Cao-Berg
Universidad Metropolitana
Nathaniel David Mercaldo
University of Idaho
Shirley Eva Sánchez
Rensselaer Polytechnic Institute
Nancy Tisch
Cornell University
Abdul-Aziz Yakubu
Howard University

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1 Abstract

Over the past five decades anthropogenic (man-made) noise has increased in the world's oceans due to an increase in shipping, oil drilling, research activities, and military explorations. All of these factors have contributed to a dramatic elevation of low frequency noise in the oceanic environment. Research on marine mammals has shown that noise below 1000Hz can cause physical trauma to their auditory system. Because of their reliance on their auditory system for survival any drastic increase in noise may compromise the survival of marine mammals. The species in the suborder Mysticeti communicate in frequency ranges from 50-1000Hz and are thus, most affected by increased noise. Six species (nearly half of the suborder) are already on the endangered species list. In order to investigate the population dynamics consequences of increased noise, we developed a system of three discrete time equations that included an explicit function for successful mating. We assume that increased low-frequency oceanic noise will reduce mating success by masking a percentage of mating calls. Analytical and numerical techniques are used to examine the long-term behavior of our system. We were able to attain thresholds for oceanic noise, which the species in question can survive.

2 Introduction

For centuries the oceans have been the sites of the amazing cacophony of marine mammal vocalizations. Symphonies of mating calls and other communications have been common sounds heard in the ocean. Unfortunately, over the past 50 years human contribution to noise has dramatically risen, as documented by the National Research Council and The Office of Naval Research [10, 16]. The dominant increase in noise has been low-frequency sound (LFS) at frequencies less then 1kHz. LFS increase can be attributed to shipping vessels, oil vessels, gas development, defense related activities, geological surveys, hydroelectric plants, and research explorations [8]. Even though many of the marine species are not at risk to increases in LFS, some marine mammals communicate at similar frequencies and therefore are likely to have disruption of effective communication [8, 12]. The marine mammals of the suborder Mysticeti (Baleen) are at an elevated risk since they communicate in frequency ranges of 50-1000Hz [10, 9].

Various types of man-made noise have been documented and observed to cause marine mammals strandings, changes in mating calls, changes of migratory patterns, and physical traumas. In 1996, thirteen Cuvier's beaked whales were found deceased on the beaches of Greece while low frequency active sonar (LFAS) was being used in the area [3]. Research on humpback whales response to LFAS showed that they slightly altered the lengths of their songs but resumed normal calling a few hours after LFAS was removed from the environment. Grey whales exposed to a test source in the middle of their migration path altered their route to steer from the source, but as soon as the noise source was removed they resumed their normal path [8]. There are many other documented cases of the negative effects of noise on marine mammal behavior [Appendix Marine Mammals and Noise].

Research on marine mammals indicates that various noise levels may have fatal effects on hearing. Multiple exposures to noise may cause temporary or permanent hearing loss that could lead to catastrophic outcomes. The Humpback Whale (*Megaptera novaeanglicie*) and the Northern Right Whale (*Eubalaena glacialis*)use sounds for contact calls, mating displays and for maintaining the cohesion of migratory herds [8], a disruption in any of these activities or behaviors may be fatal to the species survival. For marine mammals, damage to the sensory hair cells in the inner ear is permanent since they are not replaced [3, 10].

Overall it is not surprising that increased anthropogenic noise will detrimentally affect marine mammals that communicate in or near the same frequency range as the noise (i.e. the Mysticeti). Any sound present in the environment that interferes with natural communication potentially compromises the survival of mammalian life. The focus of this paper will be to investigate the population dynamic consequences of increased noise with the underlying assumption that noise will negatively effect mating success, ultimately population persistence [10].

Specifically we will examine two species of Mystieceti whales, the Finback and the North Atlantic Right Whale, both of which are already on the brink of extinction. For both species, we will analyze the population dynamics using a nonlinear discrete time model. For the North Atlantic Right Whale, we will make direct comparisons between

our non-linear model and a linear stage structured model for which parameters have been estimated from census data [3].

2.1 Mysticeti Life Cycle

The Baleen whales are constrained to rear only one offspring at a time, resulting in low reproductive rates. Accordingly, the females invest a large amount of maternal care into the offspring. There is a 10 to 12 month gestation period that is followed by a 4 to 12 month lactation period. The juvenile period varies among species but sexual maturation generally occurs within 10 to 15 years. The longevity (average lifespan of 85 years) of the Baleen whales compensates for the low reproductive rates [9, 14].

Observations of mating activity suggest that baleen whales generally mate in multi-male groups. Thus, there is a level of competitive behavior for reproductively mature and available females (without a calf). The survival of this suborder of whales is dependent on the females, since females are the only caregivers for the young, a female dominated species [7, 9, 14].

2.1.1 North Atlantic Right Whale

Eubalaena Glacialis

Population abundance estimates for this species vary from 350-700 [3, 14]. This species was the target of early whalers until their hunting was restricted in 1969 [17]. However, the species has never recuperated to a steady population level. They have been seen traveling alone, in pairs, and in groups of 5 - 10. Right whales have been observed to aggregate into mating groups, where numerous mates compete for access to an adult female. Females bare one calf per pregnancy, with a calving interval of 3.67 years, gestation period of 12-14 months and stay with calf for one year. Preliminary evidence suggests that the North Atlantic Right whale population may be steadily declining to a point where genetic variability is low, due to inbreeding. This poses a problem because as a populations genetic make-up homogenizes, the population is more susceptible to negative external fluctuations (e.g. disease) [3, 14].

2.1.2 Fin Whale

Balaenoptera physalus

The North Atlantic Whale belongs to the Balanopteridae family (rorqual whales). Population abundance estimates for the North Atlantic population is 46,000, still much below its former size. A full grown adult may weight up to 40 tons and attain a length ranging from 45-70 feet. The distinguishing characteristic of the fin whale is its dorsal fin, which is about 60cm tall, located two thirds of the way between head and tail.

Fin Whales are usually found in groups of three to pods of 10 to 20; singles and pairs are also often observed. Males reach sexual maturity between the ages of 8 to 12 ages; females between the ages of 6 to 10 years. Similar to the other baleen whales

they have one calf per pregnancy with a calving interval of 2.7 years, a gestation period of 10 to 11 months, and a lactation period of 6-7 months [9, 14, 11].

3 Models

3.1 General Mysticeti Nonlinear Model

The intricacies of the behaviors of Mysticeti whales, such as mating and reproduction, warrant the use of a nonlinear system because they accurately reflect the biological complexities that envelope the world around us than their linear counterpart. Since the vital rates vary among species of baleen whales and vary between genders, we will use a nonlinear system of difference equations to model the population dynamics and mating behavior of Mysticeti whales. Our model, Equations 1 - 3, is a special case of a set of nonlinear equations derived by Carlos Castillo-Chavez *et. al* [2]

$$x(t+1) = \beta_x \mu_m p(t-d)\mu_j^d + \mu_x x(t) + \mu_x p(t) - \phi(x(t), y(t)) \quad (1)$$

$$y(t+1) = \beta_y \mu_x p(t-d)\mu_j^d + \mu_y y(t) \quad (2)$$

$$p(t+1) = \phi(x(t), y(t)) \quad (3)$$

where $\phi(x(t), y(t)) = \frac{\mu_x x(t)\mu_y y(t)(1-\epsilon)}{\mu_x x(t)+\mu_y y(t)}$.

In our model, the single female class at generation $t+1$ is the sum of the surviving newborns, juveniles, females from the parental class, females that did not mate and females leaving the single class to the parental class in generation t. The male class at time $t+1$ is generated from the surviving newborns, juveniles and sexually mature males in generation t. Finally, the parental class at time $t+1$ is the number of females that occur a successful, yielding a calf, mating at generation t.

Notation	Definition
x	females
y	males
j	juveniles
$x(t)$	female population size at time t
$y(t)$	male population size at time t
$p(t)$	female with calves, parental class population at time t
$p(t-d)$	time delay, d, to reach sexual maturity
$\beta_{x,y}$	birth rate = 0.5
μ_m	mother survival probability
$\mu_{x,y,j}$	survival probability of females, males, and juveniles
ϵ	percentage of masked mating calls due to anthropogenic noise

Table 1: Notation

For simplicity, we will assume that $\mu_m = \mu_x$.

3.1.1 The Mating Function

This model of Baleen whale population dynamics is unlike many other models due to the incorporation of the mating function. The mating function, $\phi(x(t), y(t))$, produces the number of females that have a successful mating during one reproductive season. In our case:

$$\begin{aligned} \phi(x(t), y(t)) &= \mu_x x(t)[1 - G(x(t), y(t))] \\ &= \mu_x x(t) \frac{\mu_y y(t)(1-\epsilon)}{\mu_x x(t) + \mu_y y(t)} \\ G(x(t), y(t)) &= 1 - \frac{\mu_y y(1-\epsilon)}{\mu_x x(t) + \mu_y y(t)} \end{aligned} \tag{4}$$

where $G(x(t), y(t))$ is the probability of an unsuccessful mating season.
From Equation 4, as noise increases a proportion of mating calls are masked. This will affect the number of successful matings and hence, more unfertilized females will leave the mating grounds when the season is complete. Therefore, depending on the percentage of masked calls, the number of pairing in the next reproductive season should increase, $\phi(x, y) > 0$, when $x > 0$ and $y > 0$. For this to hold true the following conditions are placed on $\phi(x(t), y(t))$ [2]:

1)$\phi(x(t), y(t)) \geq 0$
2)$\phi(cx(t), cy(t)) = c\phi(x(t), y(t))$
3)$\phi(x, 0) = \phi(0, y) = 0$
4)$\phi_x(x(t), y(t)) \geq 0, \phi_y(x(t), y(t)) \geq 0$

3.2 Case I: Constant Mating Probability

In this case, we assume that $x(t)$ is a multiple of $y(t)$ thus producing a constant mating probability, $G((x), y(t)) \equiv K$. At a given time t, a whale population is finite, thus countable. Since the population is only composed of three groups, one group is always a multiple of the others. Intuitively, under current conditions, a steady state should be achieved. But since this is not known, $x(t)$ and $y(t)$ may not always be the same multiple of each other which we will address in Case II. Due to this constant probability, the nonlinearity of the System (1) is removed resulting in a linear system.

$$\begin{aligned} x(t+1) &= \beta_x \mu_x p(t-10)\mu_j^{10} + \mu_x p(t) + \mu_x x(t)K \\ y(t+1) &= \beta_y \mu_x p(t-10)\mu_j^{10} + \mu_y y(t) \\ p(t+1) &= \mu_x x(t)K \\ K &= \frac{\mu_y y(t)(1-\epsilon)}{\mu_x x(t) + \mu_y y(t)} \end{aligned} \tag{5}$$

To find the fixed points of this system, we solve $x(t+1) = x(t)$ and $y(t+1) = y(t)$. The scaled fixed points for the available females and sexually mature males are:

$$\begin{aligned} \frac{x(t)}{p(t)} &= \frac{\frac{\beta_x \mu_x \mu_j^{10} p(t-10)}{p(t)} + \mu_x}{1 - \mu_x K} \\ \frac{y(t)}{p(t)} &= \frac{\beta_y \mu_x p(t-10)\mu_j^{10}}{p(t)(1-\mu_y)} \end{aligned}$$

Therefore the fixed point for this system is $\left(\frac{\frac{\beta_x \mu_x \mu_j^{10} p(t-10)}{p(t)} + \mu_x}{1-\mu_x K}, \frac{\beta_y \mu_x p(t-10)\mu_j^{10}}{p(t)(1-\mu_y)}, 1 \right)$ and is stable when $|\mu_x K| < 1$ and $\mu_y < 1$. Since $\mu_{x,y} > 0$ and $0 \leq K \leq 1$, these conditions are always satisfied. To write System 5 as a system of first order difference equations, we let $q_i = p(t-i)$ for $i = 0...10$ which yields:

$$\begin{aligned} x(t+1) &= \beta_x \mu_x q_{10}(t)\mu_j^{10} + \mu_x q_0(t) + \mu_x x(t)K \\ y(t+1) &= \beta_y \mu_x q_{10}(t)\mu_j^{10} + \mu_y y(t) \\ p(t+1) &= \mu_x x(t) - \mu_x x(t)K \\ q_1(t+1) &= q_0(t) = p(t) \\ q_2(t+1) &= q_1(t) = p(t-1) \\ &\vdots \quad \vdots \\ q_{10}(t+1) &= q_9(t) = p(t-9) \end{aligned} \tag{6}$$

System (6) can be rewritten into the form of a Malthus Model, $N(t+1) = AN(t)$, where:

$$\begin{bmatrix} x(t+1) \\ y(t+1) \\ p(t+1) \\ q_1(t+1) \\ q_2(t+1) \\ \vdots \\ q_{10}(t+1) \end{bmatrix} = \begin{bmatrix} I & II \\ III & IV \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \\ p(t) \\ q_1(t) \\ q_2(t) \\ \vdots \\ q_{10}(t) \end{bmatrix}$$

and where:

$$I = \begin{bmatrix} \mu_x K & 0 & \mu_x & 0 & 0 & 0 & 0 \\ 0 & \mu_y & 0 & 0 & 0 & 0 & 0 \\ \mu_x(1-K) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, II = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \beta_x \mu_x \mu_j^{10} \\ 0 & 0 & 0 & 0 & 0 & \beta_y \mu_x \mu_j^{10} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$III = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, IV = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The solution of this linear model is $N(t) = A^t N_0$[4]. From this, a dominant eigenvalue can be obtained, as well as its associated eigenvector. From this eigenvector the stable age distribution can be determined and gives insight towards the long-term behavior of each class within the population.

3.2.1 Finback Whale Analysis

The parameter estimates of the Finback whale are: $\beta_x = \beta_y = 0.500$, $\mu_x = 0.955$, $\mu_y = 0.965$, and $\mu_j = 0.960$. For simplicity, we assumption that there is a one to one sex ratio, $K = .50402$. Using these parameters, the eigenvalues are obtained under different noise conditions:

$$\begin{aligned} \epsilon &= 0.000 \rightarrow \lambda^* = 1.030049 \\ \epsilon &= 0.655 \rightarrow \lambda^* = 1.000067 \\ \epsilon &= 0.656 \rightarrow \lambda^* = 0.999992 \\ \epsilon &= 1.000 \rightarrow \lambda^* = 0.965000 \end{aligned} \tag{7}$$

From (7), when noise is absent from the system, the dominant eigenvalue is $1.03 > 1$, thus the population slowly increases. When all the mating calls are masked the

population will go to extinction (thus $\lambda^* < 1$). There is a critical value between $(0.655, 0.656)$ when the population changes from constant to on the verge of extinction which may imply a possible bifurcation.

3.2.2 North Atlantic Right Whale Analysis

The parameter estimates of the North Atlantic Right whale are: $\beta_x = \beta_y = 0.500$, $\mu_x = 0.925$, $\mu_y = 0.940$ and $\mu_j = 0.957$. Again, a one to one sex ratio is assumed producing $K = .50260$. From simulatoins:

$$\begin{aligned} \epsilon &= 0.000 \rightarrow \lambda^* = 1.010106 \\ \epsilon &= 0.278 \rightarrow \lambda^* = 1.000027 \\ \epsilon &= 0.279 \rightarrow \lambda^* = 0.999984 \\ \epsilon &= 1.000 \rightarrow \lambda^* = 0.940000 \end{aligned} \tag{8}$$

The critical value for which this system changes from stable to unstable lies within the range of $(0.278, 0.288)$. As vital rates decrease, the percentage of masked calls envoke a greater role in the stability of the population.

3.3 Case II: Varying Mating Probability

External factors can effect populations either positively or negatively. Depending on these external factors, the vital rates, including the probability of a successful mating, is subject to waxing and waning. Our investigation is focused on how noise effects the interactions between the two sexes, therefore to address this question we will focus upon a nonlinear approach which eliminates the assumptions made in Case I. The resulting system is:

$$\begin{aligned} x(t+1) &= \mu_x \beta_x p(t-10)\mu_j^{10} + \mu_x x(t) + \mu_x p(t) - \phi(x(t), y(t)) \\ y(t+1) &= \beta_y \mu_x p(t-10)\mu_j^{10} + \mu_y y(t) \\ p(t+1) &= \phi(x(t), y(t)) \end{aligned} \tag{9}$$

where $\phi(x(t), y(t)) = \frac{\mu_x x(t) \mu_y y(t)(1-\epsilon)}{\mu_x x(t) + \mu_y y(t)}$.

To find the fixed points of this system we solve $x(t+1) = x(t)$ and $y(t+1) = y(t)$, therefore the scaled fixed points for available females and males are:

$$\begin{aligned} \frac{x(t)}{p(t)} &= \frac{\frac{\beta_x p(t-10)\mu_x \mu_j^{10}}{p(t)}}{1-\mu_x} - 1 \\ \frac{y(t)}{p(t)} &= \frac{\beta_y p(t-10)\mu_x \mu_j^{10}}{p(t)(1-\mu_y)} \end{aligned}$$

and the fixed point for this system is $\left(\frac{\frac{\beta_x p(t-10)\mu_x \mu_j^{10}}{p(t)}}{1-\mu_x} - 1, \frac{\beta_y p(t-10)\mu_x \mu_j^{10}}{p(t)(1-\mu_y)}, 1 \right)$. As with Case I, we eliminate the time delay within the system by introducing a group of

placement variables, $q_i = p(t-i)$ for $i = 0...10$, which transforms our system to:

$$\begin{aligned}
x(t+1) &= \mu_x\beta_x q_{10}(t)\mu_j^{10} + \mu_x x(t) + \mu_x q_0 - \phi(x(t), y(t)) \\
y(t+1) &= \beta_y\mu_x q_{10}(t)\mu_j^{10} + \mu_y y(t) \\
p(t+1) &= \phi(x(t), y(t)) \\
q_1(t+1) &= q_0(t) = p(t) \\
q_2(t+1) &= q_1(t) = p(t-1) \\
&\vdots \quad\quad \vdots \\
q_{10}(t+1) &= q_9(t) = p(t-9)
\end{aligned} \tag{10}$$

Likewise with the previous case, this form of the original system is an adaptation of the Malthus Model and geometric solutions are expected in the form [2]:

$$\begin{aligned}
x(t) &= \lambda^t x_0 \\
y(t) &= \lambda^t y_0 \\
p(t) &= \lambda^t p_0 \\
q_1(t) &= \lambda^t q_1(0) \\
q_2(t) &= \lambda^t q_2(0) \\
&\vdots \quad\quad \vdots \\
q_{10}(t) &= \lambda^t q_{10}(0)
\end{aligned} \tag{11}$$

Thus, System 10 can be rewritten as:

$$\begin{aligned}
\lambda x_0 &= \beta_x\mu_x q_{10}(0)\mu_j^{10} + \mu_x q_0(0) + \mu_x x_0 - \phi(x_0, y_0) \\
\lambda y_0 &= \beta_y\mu_x q_{10}(0)\mu_j^{10} + \mu_y y_0 \\
\lambda p_0 &= \phi(x_0, y_0) \\
\lambda q_1(0) &= q_0(0) \\
\lambda q_2(0) &= q_1(0) \\
&\vdots \quad\quad \vdots \\
\lambda q_{10}(0) &= q_9(0)
\end{aligned}$$

The two trivial solutions of System 9 are $((\mu_x)^t, 0, 0)$ and $(0, (\mu_y)^t, 0)$. Interpretation of the trivial solutions lead to the conclusion that if the population is composed of only one gender, then the population will decline geometrically at a rate $(\mu_y)^t$,$(\mu_x)^t$ respectively. Therefore, an investigation towards a nontrivial solution becomes biologically essential. The existence of a nontrivial solution requires $x_0 > 0$, $y_0 > 0$, and

$p_0 > 0$. To satisfy these conditions, (9) becomes:

$$\begin{aligned}
\frac{x_0}{q_0(0)} &= \frac{\beta_x \mu_x q_{10}(0)\mu_j^{10}}{q_0(0)(\lambda - \mu_x)} \quad 1 \\
\frac{y_0}{q_0(0)} &= \frac{\beta_y \mu_x q_{10}(0)\mu_j^{10}}{q_0(0)(\lambda - \mu_y)} \\
\lambda &= \phi\left(\frac{x_0}{q_0(0)}, \frac{y_0}{q_0(0)}\right) \\
\frac{\lambda q_1(0)}{q_0(0)} &= 1 \\
\frac{\lambda^2 q_2(0)}{q_0(0)} &= 1 \\
\vdots &\quad \vdots \\
\frac{\lambda^{10} q_{10}(0)}{q_0(0)} &= 1
\end{aligned}$$

In this case, the characteristic equation is $\lambda = \phi\left(\frac{x_0}{q_0(0)}, \frac{y_0}{q_0(0)}\right)$ and must be satisfied for a nontrivial solution to exist. Following Proposition 1 from Castillo-Chavez [2], to determine the conditions for which this equation is satisfied, let $L(\lambda) = \lambda$ and $R(\lambda) = \phi\left(\frac{x_0}{q_0(0)}, \frac{y_0}{q_0(0)}\right)$. The components of the mating function are positive if $\frac{\beta_x \mu_x q_{10}(0)\mu_j^{10}}{q_0(0)(\lambda - \mu_x)} > 1$, and $\lambda > \mu_{x,y}$. When $\lambda \to \infty$, $L(\lambda)$ is strictly increasing while $R(\lambda)$ is strictly decreasing. For a nontrivial solution to exist, an interval of λ must be computed for $L(0) < R(0)$. For our system, this interval is:

$$\left(\frac{\beta_x \mu_x q_{10}(0)\mu_j^{10}}{q_0(0)} + \mu_x > \lambda > \mu_y\right) \tag{12}$$

Since the long-run population behavior is in question, we will look at what happens when one gender goes to ∞. Since $\mu_y > \mu_x$, we will focus upon $y \to \infty$. Due to rescaling and using a Taylors series expansion of $\phi(x, y)$, this yields:

$$\begin{aligned}
\phi(x, y) &= y\phi\left(\frac{x}{y}, 1\right) \\
&\approx y\left[\frac{x}{y}\phi_x(0,1) + \phi_y(0,1)\right] \\
&\approx x\phi_y(0,1) \\
&\approx \left(\frac{\beta_x \mu_x q_{10}(0)\mu_j^{10}}{q_0(0)(\lambda - \mu_x)} \quad 1\right)\phi_y(0,1)
\end{aligned}$$

As $\lambda \to \mu_y^+$, a nontrivial solution will exist, and be of the form $((\lambda^*)^t x_0, (\lambda^*)^t y_0, (\lambda^*)^t p_0)$, if [[2]):

$$\left(\frac{\beta_x \mu_x q_{10}(0)\mu_j^{10}}{q_0(0)(\mu_y - \mu_x)} - 1\right)\phi_x(0,1) > \mu_y \tag{13}$$

where λ^* = dominant eigenvalue.
To determine the stability of this solution, let $\xi(t) = \frac{x(t)}{p(t)}$, $\eta(t) = \frac{y(t)}{p(t)}$, $\alpha(t) = \frac{p(t-10)}{p(t)}$ and $\varsigma(t) = 1$. Then in the new variables, System 9 becomes:

$$\begin{aligned} \xi(t+1) &= \frac{\beta_x \mu_x \mu_j^{10} \alpha(t) + \mu_x + \mu_x \xi(t)}{\phi(\xi(t), \eta(t))} - 1 \\ \eta(t+1) &= \frac{\beta_y \mu_x \mu_j^{10} \alpha(t)}{1 - \eta(t)} \\ \varsigma(t+1) &= 1 \end{aligned} \tag{14}$$

Let $(\xi_0, \eta_0, 1) = (\frac{x_0}{p_0}, \frac{y_0}{p_0}, 1)$ be a fixed point of System 9 and the corresponding Jacobian is:

$$J(\xi_0, \eta_0, 1) = \begin{bmatrix} \frac{\phi(\xi_0,\eta_0)\mu_x - (\beta_x \mu_j^{10}\alpha(t) + \mu_x + \mu_x \xi_0)(\phi_\xi(\xi_0,\eta_0)}{\phi(\xi_0,\eta_0)} & -\frac{(\beta_x \mu_j^{10}\alpha(t) + \mu_x + \mu_x \xi_0)(\phi_\eta(\xi_0,\eta_0)}{\phi(\xi_0,\eta_0)} & 0 \\ 0 & \frac{\beta_y \mu_x \mu_j^{10}\alpha(t)}{(1-\eta_0)^2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The eigenvalues of this matrix are:

$$\begin{aligned} \lambda_1 &= 0 \\ \lambda_2 &= \frac{\phi(\xi_0, \eta_0)\mu_x - (\beta_x \mu_j^{10}\alpha(t) + \mu_x + \mu_x \xi_0)(\phi_\xi(\xi_0, \eta_0)}{\phi(\xi_0, \eta_0)} \\ \lambda_3 &= \frac{\beta_y \mu_x \mu_j^{10}\alpha(t)}{(1-\eta_0)^2} \end{aligned} \tag{15}$$

Since $\lambda_1 = 0$, the Jacobian can rescaled to:

$$\begin{bmatrix} \frac{\phi(\xi_0,\eta_0)\mu_x - (\beta_x \mu_j^{10}\alpha(t) + \mu_x + \mu_x \xi_0)(\phi_\xi(\xi_0,\eta_0,1)}{\phi(\xi_0,\eta_0)} & -\frac{(\beta_x \mu_j^{10}\alpha(t) + \mu_x + \mu_x \xi_0)(\phi_\eta(\xi_0,\eta_0)}{\phi(\xi_0,\eta_0)} \\ 0 & \frac{\beta_y \mu_x \mu_j^{10}\alpha(t)}{(1-\eta_0)^2} \end{bmatrix} \tag{16}$$

The determinant and trace of (16) are:

$$\begin{aligned} determinant &= \frac{\phi(\xi_0, \eta_0)\mu_x - (\beta_x \mu_j^{10}\alpha(t) + \mu_x + \mu_x \xi_0)(\phi_\xi(\xi_0, \eta_0)}{\phi(\xi_0, \eta_0)} \frac{\beta_y \mu_x \mu_j^{10}\alpha(t)}{(1-\eta_0)^2} \\ trace &= \frac{\phi(\xi_0, \eta_0)\mu_x - (\beta_x \mu_j^{10}\alpha(t) + \mu_x + \mu_x \xi_0)(\phi_\xi(\xi_0, \eta_0)}{\phi(\xi_0, \eta_0)} + \frac{\beta_y \mu_x \mu_j^{10}\alpha(t)}{(1-\eta_0)^2} \end{aligned}$$

From the Jury test, $(\xi_0, \eta_0, 1)$ is asympotically stable if [2]:

$$|trace(J(\xi_0, \eta_0, 1)| < 1 + determinant(J(\xi_0, \eta_0, 1)) < 2$$

From this inequality, the long term behavior of our model can be determined and is established through a series of simulations.

4 Reproductive Disturbance by Noise: Simulations

4.1 Discussion of Simulation Code

In our model ϵ represents the proportion of unsuccessful mating calls(or masked mating calls), which we are assuming corresponds to oceanic noise levels. Programs to simulate our system with a time delay and varying ϵ were constructed in MatLab. With these routines we were able to plot population sizes, population proportion for females, males, and parental females within the population for varying ϵ and constant ϵ for each yearly interval.

The routines can use any initial condition sets (x_0,y_0,p_0), vital rate sets (μ_x, μ_y, μ_m, μ_j, β_x, β_y) and initial population sizes at each delay stage $p(t-d)$. Particular initial conditions for the two species under consideration were used for the simulations (these values were taken from current literature on the species). We are assuming that there will always be enough males to fertilize the females, due to the one sex ratio in our two species of Baleen whales [3, 13]. At the same time we are assuming there are more single females than paired females with calves, that is $x_0 > p_0$ for any simulation for our model. As we run this program with time delays, we set the population in each stage equal to each other (the survival probability is applied when the whale enters the x or y class as μ_j^{10} since it takes 10 years to reach the sexual maturation to enter those classes).

4.1.1 North Atlantic Right Whale: Numerical and Simulation Analysis

In Caswell's paper [3] he mentions a couple of key demographic characteristics of North Atlantic Right Whale.

1. About 300 NARW are observably left, with 150 females and 150 males, a 1:1 sex ratio

2. 0.38 of the female population is reproductively active, then, 0.62 of the female population is single.

Thus, initial condition set (x,y,p) can then be renamed (0.62y,y,0.38y) (recall x+p=y). These proportions allow us to pick biologically significant initial conditions for the single female population, x, the male population,y, and the parental population, p.

We calculated two initial condition sets, (93,150,57) and (186,300,114), with respect to the above definition of x and y. Simulations with either initial condition set vs. ϵ, yields population extinction. The proportion of males in the population reaches 1, that is the population is comprised fully of males, biologically this is can be interpreted as extinction. This result is also biologically realistic since males have longer life spans then females, and thus will the last survivors.

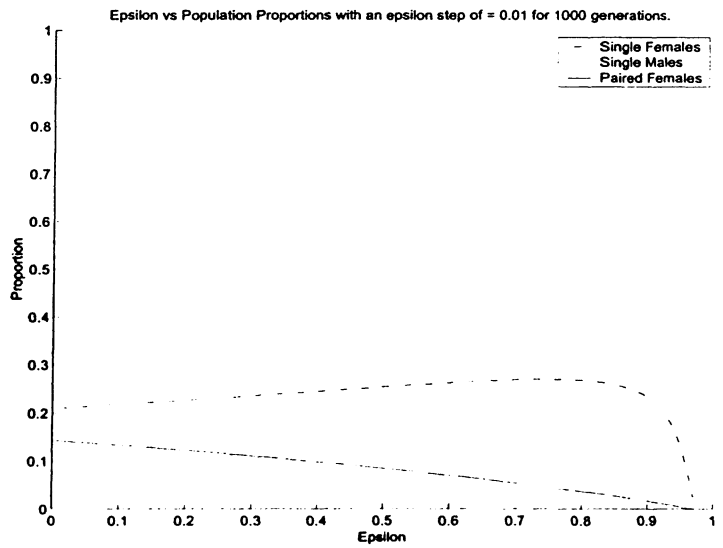

Figure 1: Path to Extinction with Increased Noise

An ideal oceanic environment for the NARW would be one without anthropogenic noise, we consider this as a simulation with $\epsilon = 0$. (Every mating call that is sent out will be received.) This simulation depicts population survival and eventual steady state proportion of 0.21 females, 0.65 males, and 0.14 parental females. Overall the population tends to increase without bound. Thus even though the initial conditions depicted an equal number of females to males, the population tends to such steady states.

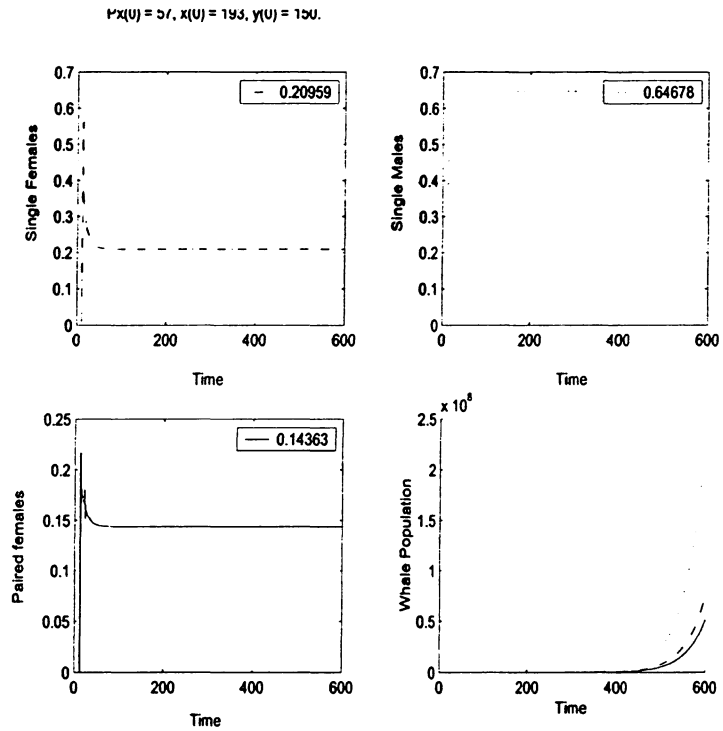

Figure 2: Population Proportions with constant $\epsilon = 0$

Another simulation with initial condition (93,150,57) and constant $\epsilon = 0.3$ produced population explosion. Most interestingly such a simulation yields steady state population proportions of: 0.236 females, 0.6525 males, and 0.1144 parentals. These steady state values are common for any set of initial conditions as we have a simulation below which randomly picks 50 sets of initial conditions, and we observe erogodicity in our system.

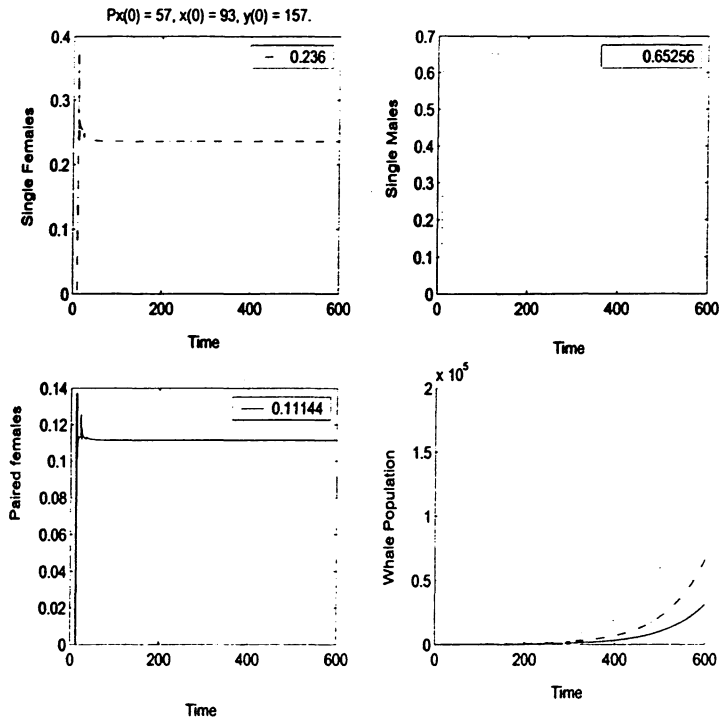

Figure 3: Population Proportions with constant ϵ =.3

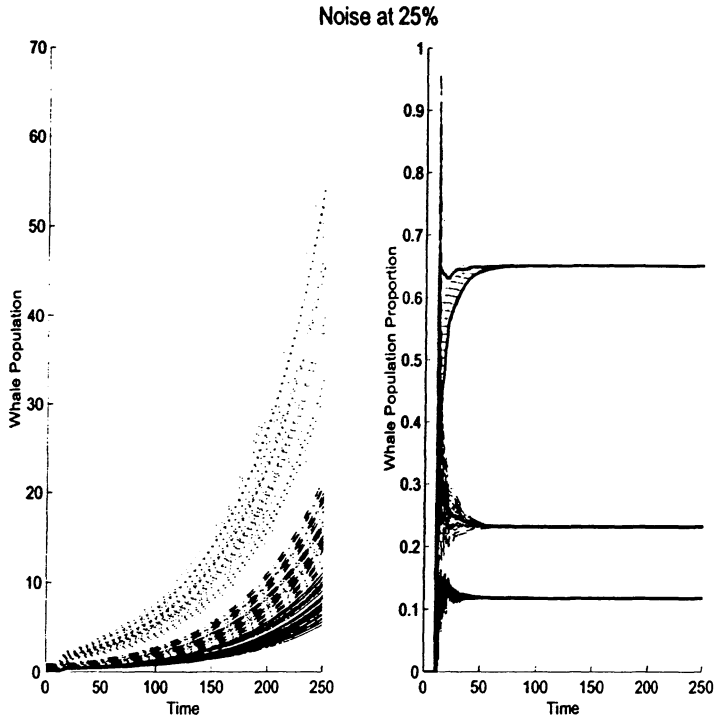

Figure 4: Population Dynamics with random initial conditions and $\epsilon = .25$

The NARW have a fertility rate of 0.19, which in our nonlinear model can be interpreted as the mating function, $\phi(x(t), y(t))$ multiplied by the number of females in the population. Thus we can calculate the ϵ value corresponding to NARW population [3].
$\mu_x = 0.925$
$\mu_y = 0.94$
(x,y,p) = (0.62y,y,0.38y)
$\phi(x, y) = \frac{\mu_x x(t)\mu_y y(t)}{\mu_x x(t)+\mu_y y(t)}(1-\epsilon)$
Evaluating our ϕ function with the vital rates we have, yields the following y dependent linear equation for ϵ.

$$\epsilon = 1 - \frac{\phi x}{0.35619y} = 1 - \frac{\phi .62y}{.35619y} \tag{17}$$

choosing y=150 and ϕ=0.19, then ϵ=0.669278
Simulations with ϵ=0.669278 as a constant during the entire simulation of 600 years yields low population levels which can be interpreted as extinction around after 200 years.

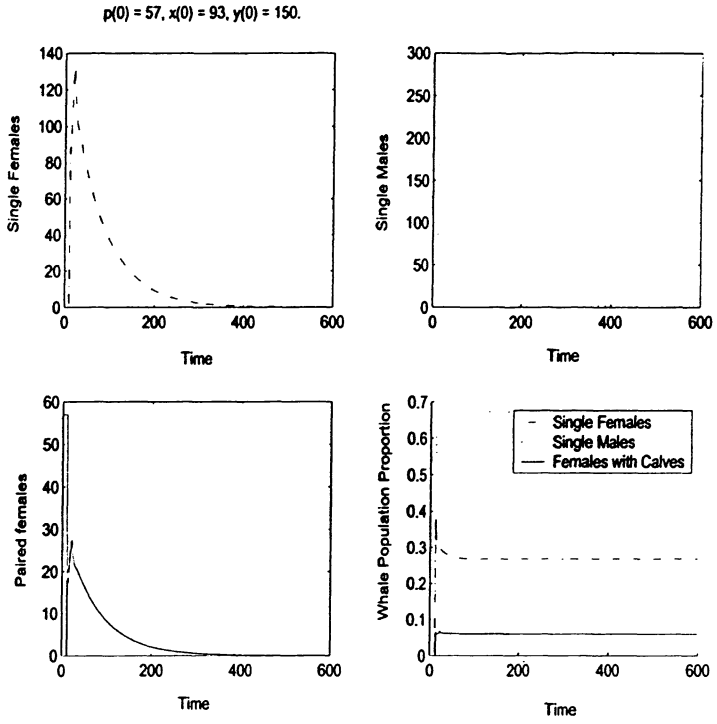

Figure 5: Population Dynamics with constant ϵ =.669278

4.2 Finback Whale: Numerical and Simulation Analysis

Parameter values used to run the simulations for the Fin whale were estimated from the NARW rates and citations [9, 11]. Considering that this species has population estimates of 4600 it probably has higher distribution of single females then parental females[15]. We assumed 40 percent of the female population are reproductively active and 60 percent are in the single class [9].

Simulation with (840,1400,560) and ϵ from 0 to 1 yield a similar graphic as the NARW, extinction as $\epsilon \rightarrow 1$. For constant noise, $\epsilon = 0.3$ we attain population explosion and steady state population proportions of: 0.25 females, 0.63 males, and 0.12 parentals, similiar behavior as the for NARW. We found a threshold for population persistence of $\epsilon \leq .751$. The figure below depicts the population slow growth.

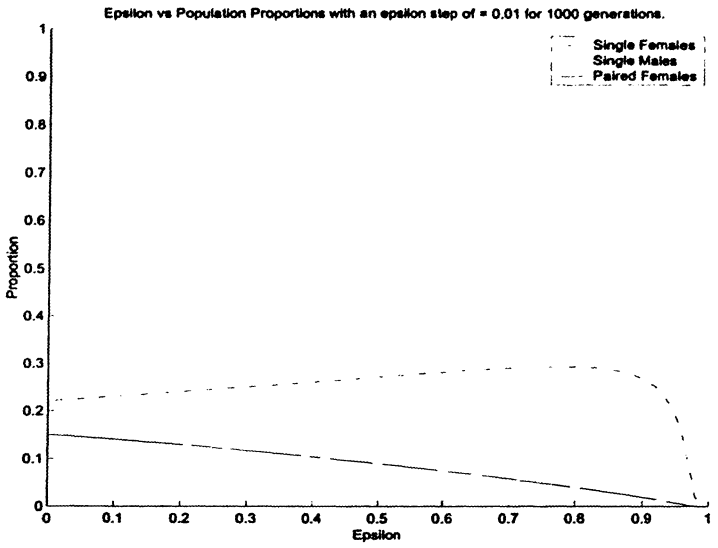

Figure 6: Path to Extinction with Increased Noise

These steady states values were attained for any $\epsilon < 0.755$ and any initial condition. The biological importance of such steady states is that the population aims to attain such a steady states regardless of initial condition to sustain a healthy and growing population (this was verified with a random initial condition program). The population with aϵ=.75 (or 75 percent of mating calls masked), begins to depict slow population growth. At this level of noise the population is just about at the tipping point at which growth or extinction may occur. For $\epsilon \gg .755$ population extinction occurs. %subsubsection Oscillatory Noise In this section we will run simulatiion that depict noise as an oscillatory function, the following are the two form they will take.

$$\phi(x(t), y(t)) = \frac{\mu_x x(t)\mu_y y(t) 0.5\left(1-(-1)^t epsilon\right)}{\mu_x x(t)+\mu_y y(t)}.$$

$$\phi(x(t), y(t)) = \frac{\mu_x x(t)\mu_y y(t) 0.5(1-\epsilon sin(2\pi)}{\mu_x x(t)+\mu_y y(t)}.$$

The first form we will refer to as time varying oscillations and the second we will refer to as sinusoidal oscialltions. Through our simulations we have observed that population extinction for any ϵ value for our sinusoidal function and a disttinct threshold for the sinusoidal function.

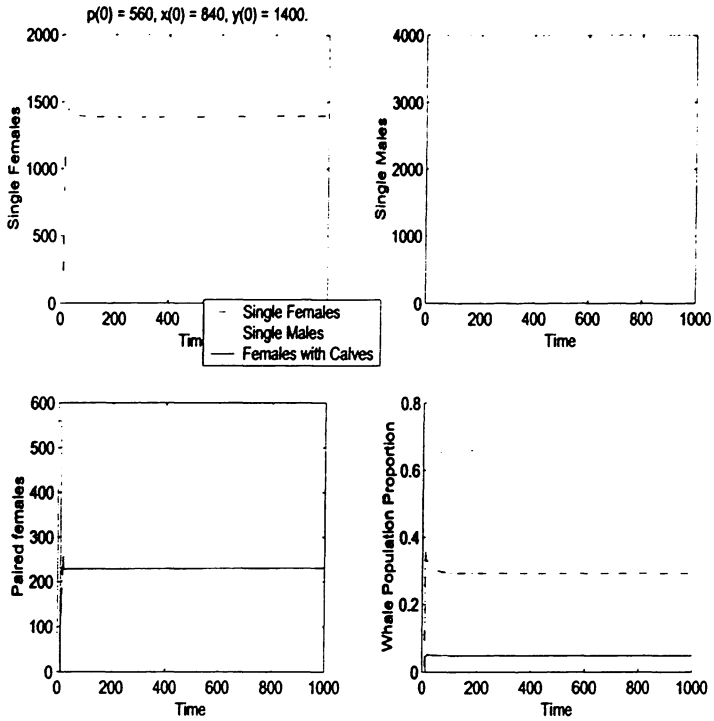

Figure 7: Population Dynamics with constant ϵ =.751

This graph shows oscillatory noise due to our time varying noise function. With this type of noise oscillations we see extinction for initial noise greater then 0.1225 and less then 0.1225 explosion. This conclusion may seem opposite to intuition, however,if observe the oscialltions for the top left graph (single females) we see that their population is bounded above $(1+\epsilon)/2$ and bounded below by $(1-\epsilon)/2$.

4.2.1 Conclusion to Simulations

From the computer simulations it is obvious to see that as ϵ (noise or reproductive disturbances) increases the populations are set on a path to extinction. Further, our model was able to produce similar extinction time for the North Atlantic Right Whales, 200 to 250 years as Caswell. However, we were able to observe the threshold for masking of the mating calls as 0.755 for fin whales and levels above such a threshold lead to extinction.

5 Conclusion

Based on this model, if noise is assumed to affect a percent of mating calls, then the entire population may be in grave danger. As new technological advances become implemented, the harmony that nature once exhibited is dwindling before our eyes. Such advances include the demand for security or for oil and offset many environmental

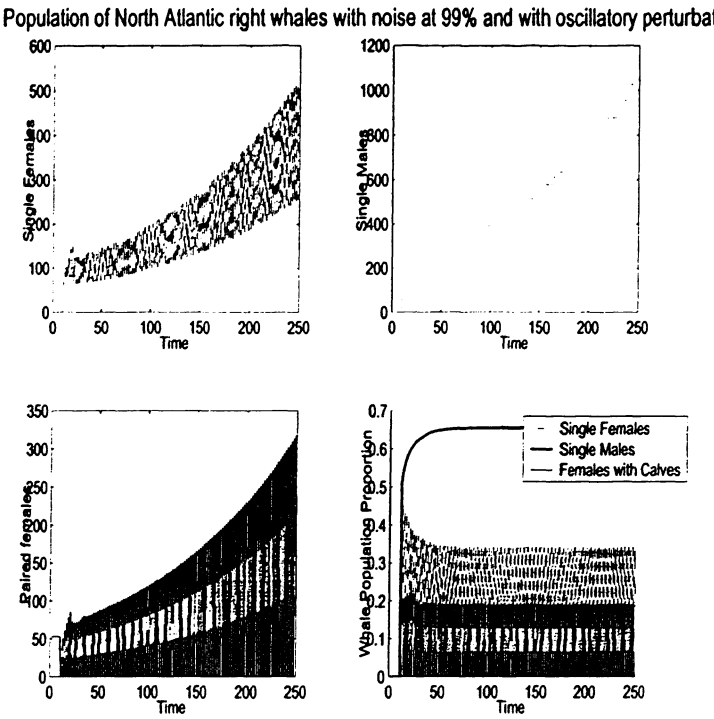

Figure 8: Population Dynamics with oscillatory noise

concerns. Due to this, the world's oceans are becoming infected with man-made noise. Therefore, we have found that the amount of noise within the oceanic environment dictates if a population explodes, remains constant or declines to extinction.

When our system is subjected to a constant noise, below a threshold value (ϵ=0.475for NARW and ϵ =0.751 for the fin whales), the population reaches an ergodic stable stage distribution. This occurs when an amount of noise is applied that does not greatly exceed that of the normal background noise of the ocean, thus normal communication between whales is not heavily altered. When the amount of noise exceeds this threshold value the population declines to extinction because the potential of physical trauma increases and a majority of the mating calls are unsuccessful.

Not only does the amount of noise affect the dynamics of a population, but also how the noise is distributed. It was shown that under constant noise, a threshold value was obtained such that when this value was reached the population was doomed to extinction. When the noise varies over time, such as using a sinusoidal function the population always expires, but at an unhurried rate than that of constant noise. When the level of noise varies equally between the extremes, the threshold value is violated. Due to this, the population may suffer tremendous losses if the time spent with the increased noise is any significant length. But after reaching this maximum noise level, the level begins to decline and the remaining individuals begin to procreate which restores a portion of the original population. Since the lifespan of the Mysticeti whales range up to 100 years, the replenishing time of the population is never reached, thus

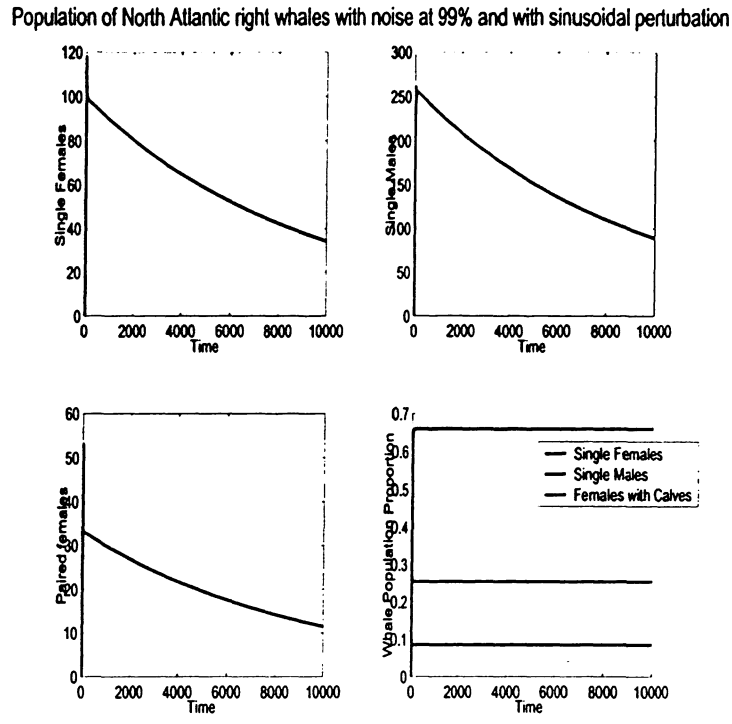

Figure 9: Sinusoidal Noise

driving the entire remaining population to extinction.

Based on these findings, many beneficial implications could arise. Such environmental policies that would limit the noise length and level at which certain industrial activities operated would greatly reduce the anthropogenic noise within the oceans. With a reduction in the anthropogenic noise the overall health and abundance of numerous marine mammals, not only Baleen whales, would greatly increase. Further through our oscillatory form of noise it is evident that distribution of noise over, can greatly effect the long term dynamics of our population. Biologically this occurrence is realistic, since it means that as the rate of masking mating calls increase, the population size decreases. Our model does not account for environmental carrying capacity that of course will restrict population explosion. However, it does give us a good glimpse of the behavior of the population in response to reproductive disturbances, which w we consider to be primarily noise.

5.1 Future Work

Many simplifications have made in our model due to the lack of current information on Baleen whales and thus with more time we would have many directions to further our research. We would like to continue our research and incorporate the effect of noise in the survival/mortality (and other vital rates) rates of our system. We would also like to create a separate juvenile class that would eliminate the time delay; it would make

our model more exact. A phenomena that has been observed in the North Atlantic Right Whale has been an increase in the calving interval due to environmental stress [3], we would like to incorporate such a delay into our model. We would also like to work with more realistic ϵ and develop some way to interpret oceanic noise as it relates to reproductive success. The approaches to how to model noise distribution over the projection intervals have been theoretical, we like to model the noise fluctuation in the past and use the model to project our future population.

5.2 Acknowledgments

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6 Appendix: Motivation for Nonlinear Models

The following model, for a female dominant species, has been derived from a nonlinear mating model by Carlos Castillo-Chavez(2001) [2]Various changes of the initial model have been made according to the biology and demographics of our species of Baleen whales. The first change that has been made, which simplified our model from a system of four equations to that of a system of three equations, was that within the social structure of Finback whales species is a female dominated species [?]. Biologically this means that the females are the ones that care for the young, thus the male-partnered class can be deleted. Further, the remaining female partnered class is renamed the parental female class. In our model there is no proportion of the female class that is widowed or separated, since the males and females only come together during the winter months to mate. [4, 9, 14]

A nonlinear mating model stimulated the construction of our nonlinear models. Though much of dynamics of the this model did not apply to our female dominant model of whale mating, the analysis and form of this model served as a starting point of what demographic dynamics to look at in our species of marine mammals. Nonlinear Mating Models for Populations with Discrete Generations by Carlos Castillo-Chavez, Adbul-Aziz Yakubu, Horst Thieme, Maia Martcheva (March 21,2001) was the source of the original system of equations and theory behind much of the ana-

lytical developments of this paper, but were adapted to fit the biology of the Baleen whales.

$$
\begin{aligned}
x(t+1) &= (\beta_x\mu_x\mu_y + (1-\mu_x)\mu_y + (1-\sigma)\mu_x\mu_y)p(t) + \mu_x x(t) - \rho_G(x(t), y(t), p_x(t), p_y(t)) \\
y(t+1) &= (\beta_y\mu_x\mu_y + (1-\mu_x)\mu_y + (1-\sigma)\mu - x\mu_y)p(t) + \mu_y y(t) - \rho_H(x(t), y(t), p_x(t), p_y(t)) \\
p_x(t+1) &= \sigma\mu_x\mu_y p(t) + \rho_G(x(t), y(t), p_x(t), p_y(t)) \\
p_y(t+1) &= \sigma\mu_x\mu_y p(t) + \rho_H(x(t), y(t), p_x(t), p_y(t))
\end{aligned}
$$

where $\rho_G(x(t), y(t), p_x(t), p_y(t)) = \mu_x x(1-G(x, y, p_x, p_y))$ and $\rho_H(x(t), y(t), p_x(t), p_y(t)) = \mu_x x(1 - G(x, y, p_x, p_y))$

6.1 Appendix: Marine mammals and Noise

Marine life has been continuously been disturbed by increased noise. Documentation of the temporary effects of anthropogenic noise on cetaceans includes longer dive times, shorter surface intervals, evasive movements away from the sound source, attempts to shield young, increased swimming speed, changes in song note durations and departure from the area [8]. Industrial noise has also been known to effect marine life, studies have found the following effects: (1) migrating gray whales (Eschrichtius robustus) exhibited an 80 percent avoidance reaction to oil exploration sounds played at 130dB from a sound source directly in their migration path, (2)migrating gray whales exhibited a 10 percent avoidance response to air gun sounds played from a source directly in their migration path(3)Bowhead whales avoided seismic exploration activities at ranges of 2 Km and 20 Km (4) sperm whales stopped vocalizing in response to weak seismic pulses from a distant ship. The following are some more examples of behavioral changes of marine life in response to noise: (1) sperm whale cessation of activities and scattering away from sonar signals between 3.25 and 8.4 kHz, (2) increased stranding of beaked whales associated with the time of military operations (Simmonds an Lopez-Jurado, 1991)(3)cessation of sperm whale echolocation clicks in reaction to an acoustics themography sound source (4) a shift in distribution of humpback whales and sperm whales away from the low-frequency sound source when it was transmitting [8].

6.2 Appendix: Establishment of Parameters

In the research paper, Declining survival probability threatens the North Atlantic Right whale [3], Dr Caswell constructs a matrix population model of NARW using sighting data (>10,000) of photographically identified individuals since 1980. Estimates of the sightings were derived to analyze the causes of the right whale imperilment. An important observation of the species is that the calving interval has increased from 3 years 1985 to 5 years in 1990.

The asymptotic population growth rate, λ_t, was calculated.

Criteria:

$\lambda_t > 1$ population exhibits exponential growth at time t(surv ival/explosion of population)

$\lambda_t < 1$population exhibits exponential decay (extinction)
λ_{1980} was calculated to be 1.03 and $\lambda_{1995} = .98$, thus over a fifteen year span the population has truly entered a path to extinction. An LTRE (Life Table Response Experiment) analysis was done on this model to verify which vital rates most influence the survival of the species. It must be observed that increased calving intervals and increased mortality rates were two factors that have greatly affected the NARW species from the 1985 to 1990. Caswell's conclusion is that the decreases of the survival probabilities, specifically of the mothers, have been the major factors driving NARW to extinction. He speculated that if at lease two females deaths could have been prevented every year since 1985 to 1990, λ_{1990} remained greater then 1. From, Casewell's life cycle model of NARW we were able to attain the vital rates necessary for our nonlinear model.

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